

Study of coupled bunch instabilities in the CERN SPS

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- Motivation for studies
- Effect of instabilities on the beam
- Cures
- Search for sources
 - analysis of spectra

Results of measurements done with
T.Bohl, T.Linnecar and J.Tuckmantel

With help from CERN PS
SL/OP,
R.Cappi, R.Garoby, D.Manglunki, ... (PS)
in preparing different cycles and beams

Future use of high intensity proton beams in the CERN SPS

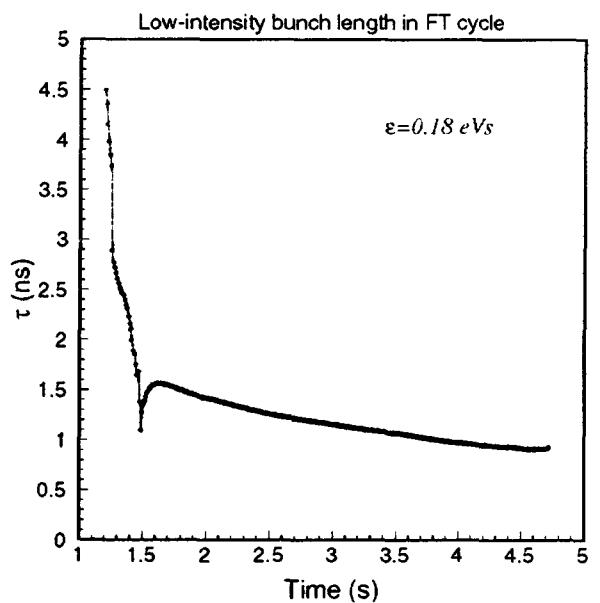
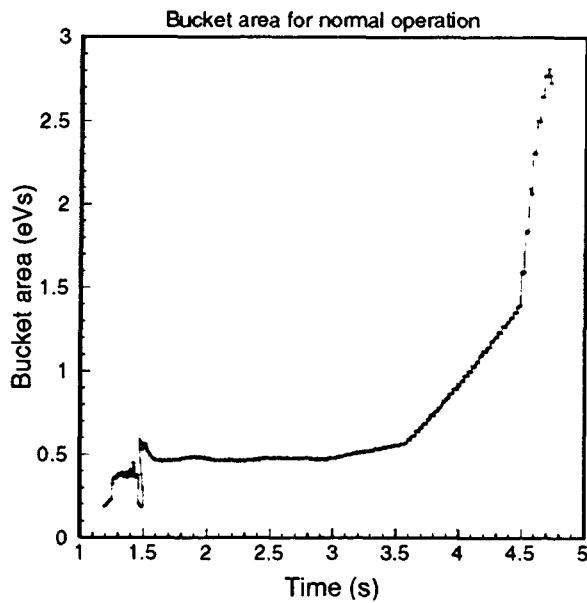
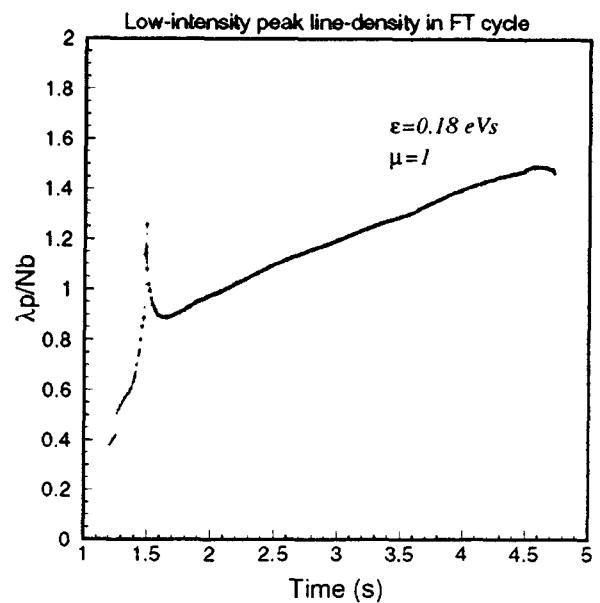
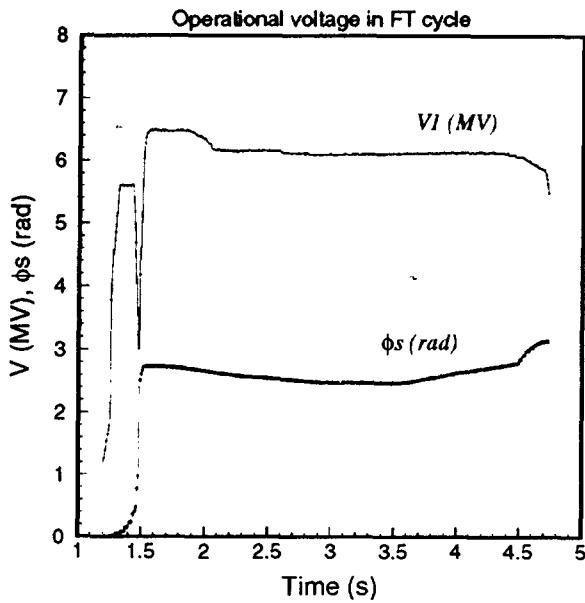
- Injector for LHC
 - high bunch intensity (up to 2×10^{11}) in multi-bunch mode of operation (243 bunches)
- Production of neutrino beams
 - high total intensity ($> 5 \times 10^{13}$)

Parameters of fixed target (FT) and LHC beam

Beam	FT	LHC
injection energy (GeV)	14	26
extraction energy (GeV)	450	450
transition crossing	yes	no
RF system (MHz)	200	200
bunch spacing (ns)	5	25
filling pattern	10/11	3/11
number of bunches	4200	243
intensity/bunch	10^{10}	10^{11}
total intensity	4.3×10^{13}	2.5×10^{13}
long. emit. at inj. (eVs)	0.18	0.35
long. emit. at ext. (eVs)	?	0.5-1.0

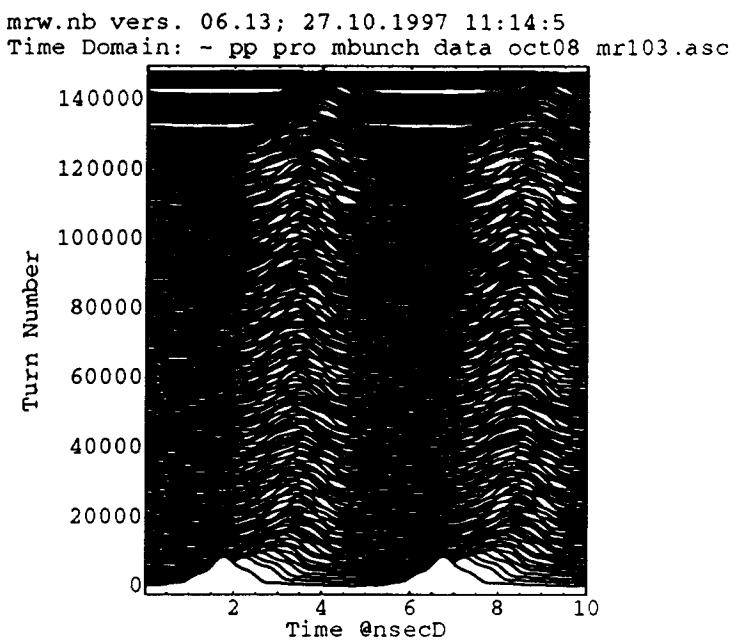
Fixed target cycle in the SPS

(normal operation)

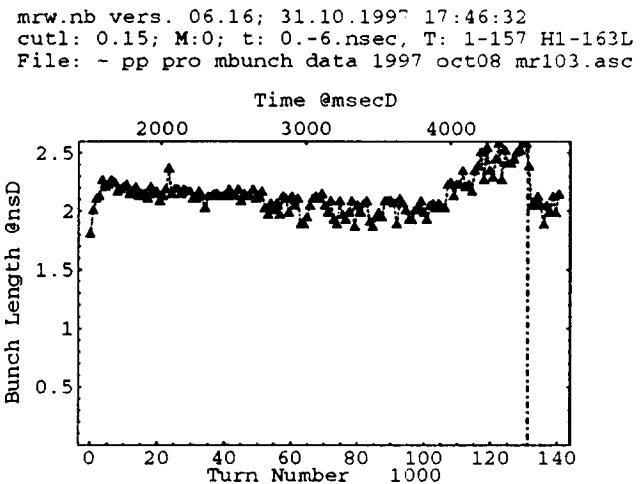


Measurements on fixed target cycle

Mountain range display



Bunch length (at 0.85 of the peak line density)



Beam intensity 4.2×10^{13} in 2 batches (10/11)

Measurements with modified voltage programmes

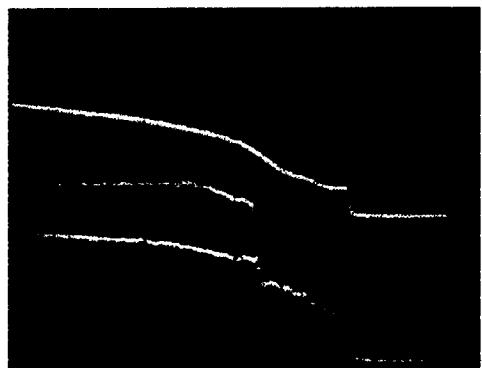
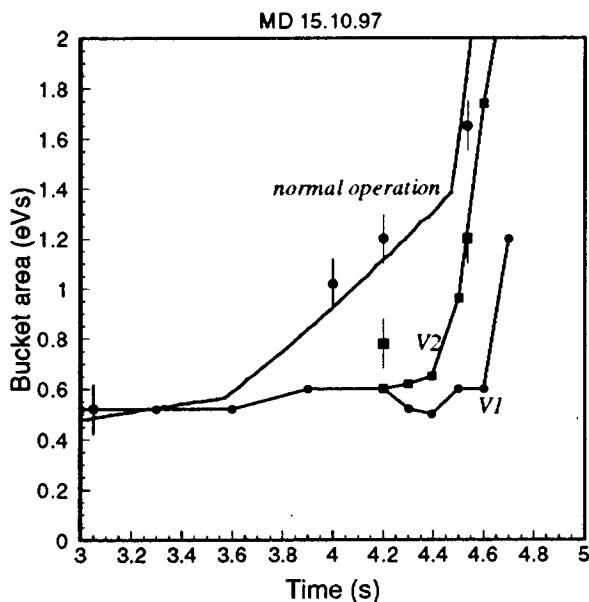
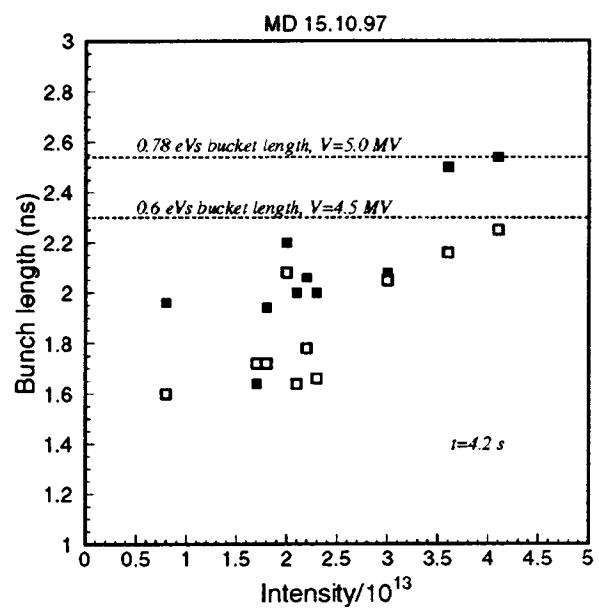
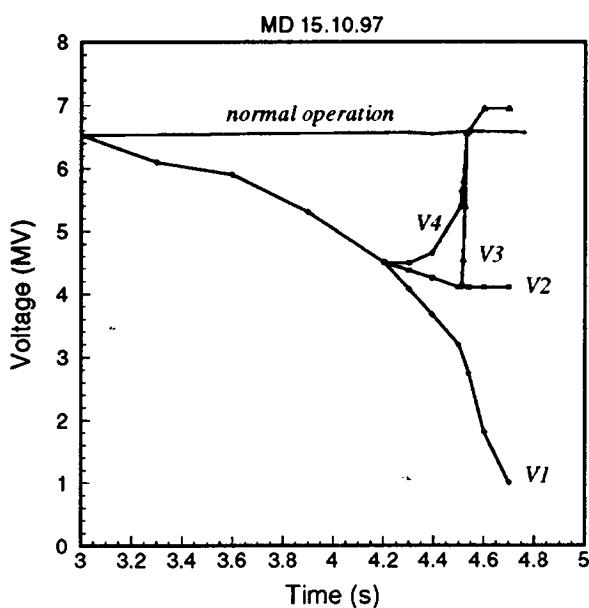
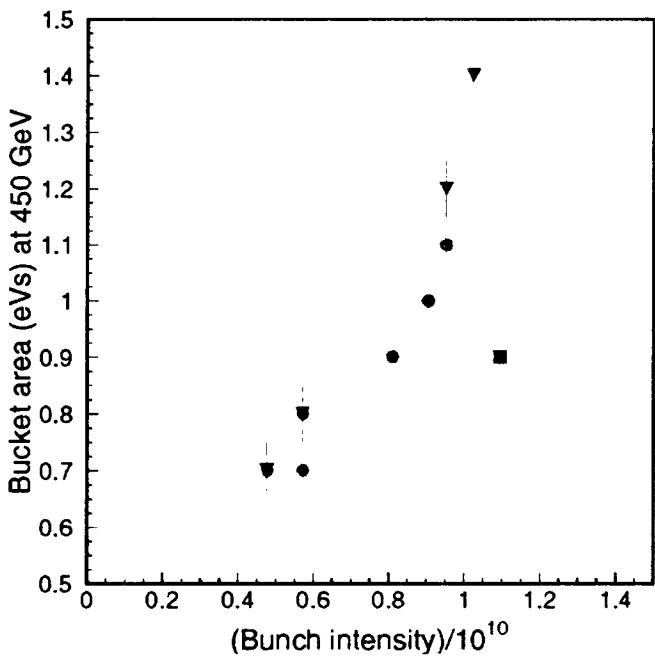
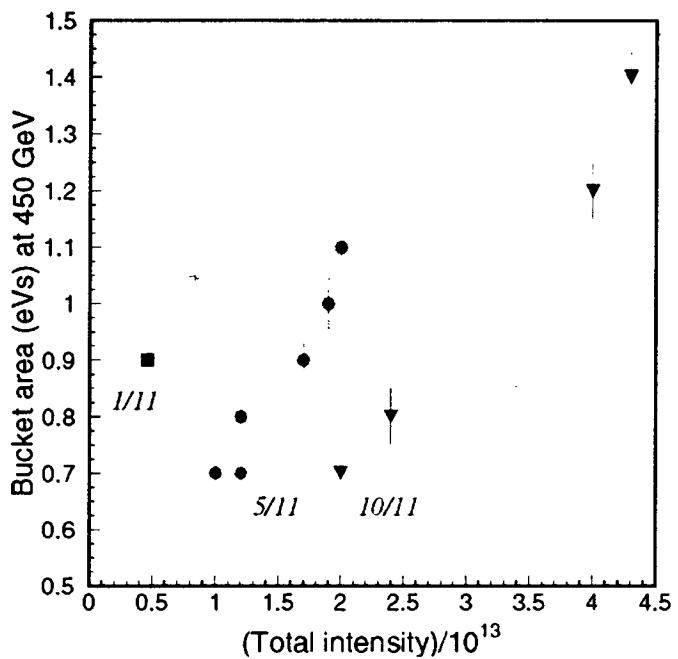


Photo: top - voltage V1, middle - beam current, bottom - peak detected signal, beam intensity 3.0×10^{13} , trigger 4.0 s, 0.1 s/div

V3 voltage programme: $\varepsilon = 1.3 \text{ eVs}$, V4: $\varepsilon = 1.6 \text{ eVs}$

Measurements for full and partially filled SPS rings
Bucket area for no observed losses at the end of the cycle
in 1/11, 5/11 and 10/11 filled ring



Effect on the beam - summary:

- In normal high-intensity operation longitudinal emittance blows-up by factor 10: 0.2 eVs → 2.0 eVs.
- Modified voltage programmes (decreased voltage) give emittance of 1.2 eVs (@445 GeV).
- Attempt to minimise the bunch length before extraction leads to further blow-up.
- In the first approximation emittance blow-up is defined by intensity per bunch and less by total intensity.
- For larger gap in the ring beam is more stable. (Indication for low Q impedance?)
- Things are getting worse towards the end of the cycle.

Why things are getting worse towards the end of the cycle?

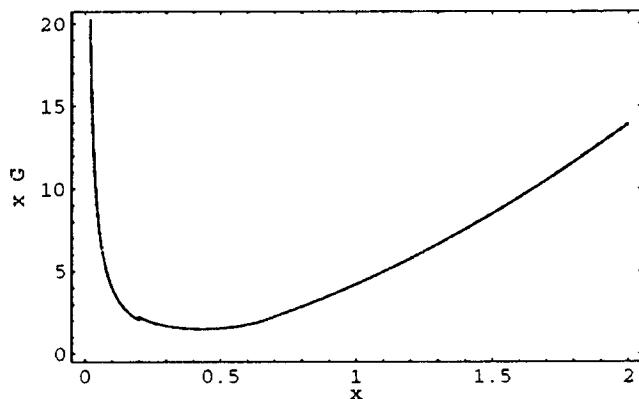
Threshold for coupled-bunch instability due to resonant impedance with frequency

$$\omega_r = 2\pi f_r = (lM + n)\omega_0 + m\omega_s$$

$$R_{sh} < \frac{|\eta|E}{eJ_A} \left(\frac{\Delta p}{p} \right)^2 \frac{\Delta\omega_s}{\omega_s} \frac{F}{f_0\tau} xG(x),$$

$\omega_0 = 2\pi f_0$ is the revolution frequency, $\eta = 1/\gamma^2 - 1/\gamma_t^2$, J_A is the average beam current, $\frac{\Delta p}{p}$ is the relative momentum spread, $\frac{\Delta\omega_s}{\omega_s}$ is the relative synchrotron frequency spread, τ is the bunch length, the formfactor $F \sim 0.3$ is defined by the particle distribution.

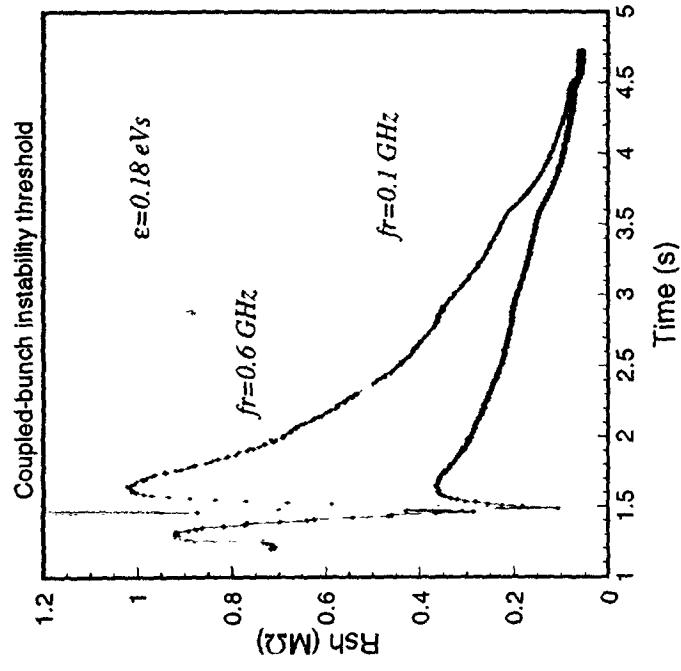
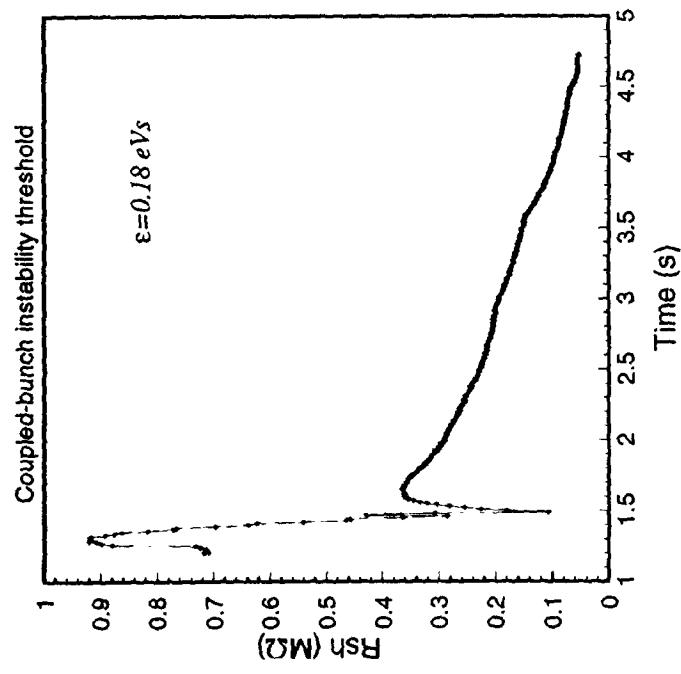
Function $xG(x) = x \min\{J_m^{-2}(\pi x)\}$, $x = f_r\tau$



$f_r\tau$

Limitation on R_{sh} for coupled-bunch instability during fixed target cycle, $N = 4.2 \times 10^{13}$

For $xG = (xG)_{min} = 1.5$ For low and high frequency impedances

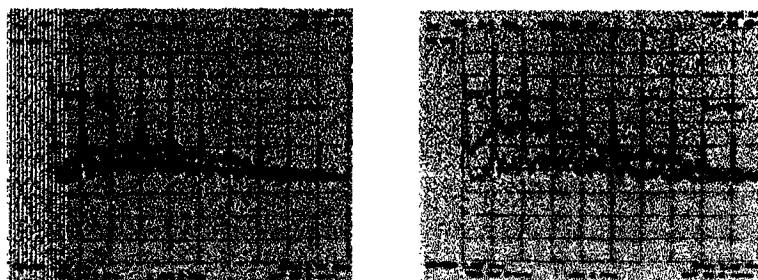


Example:

Spectrum observed for low intensity beam with $N = 4 \times 10^{12}$ (MD April 1998)

Cures

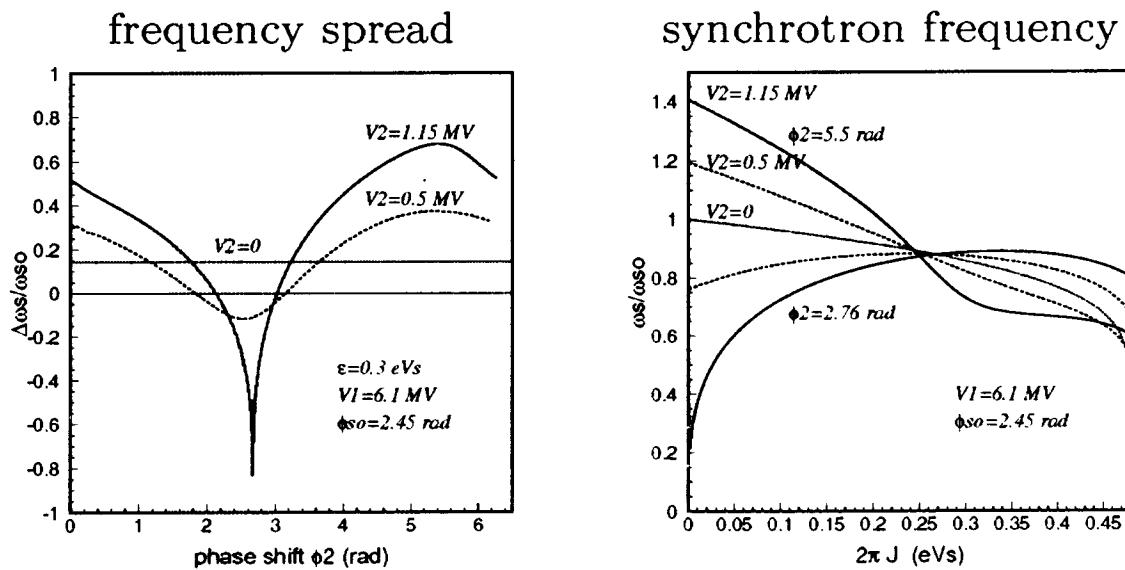
- Search for isources → improved passive damping
- Active damping using feedback system
- Landau damping using high (4-th) harmonic RF system (800 MHz) in bunch shortening (BS) mode



Beam spectrum from 0 to 2 GHz

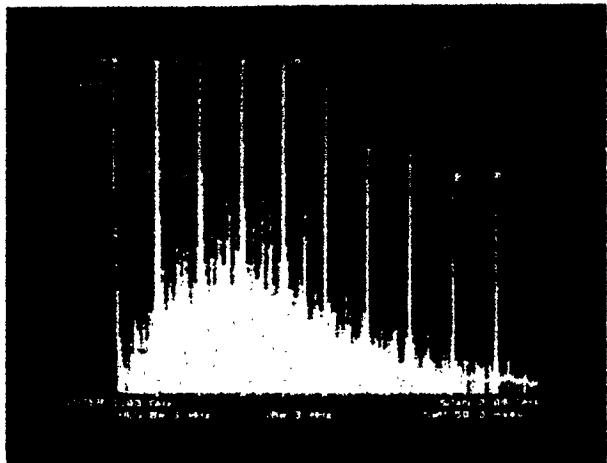
with 800 MHz RF system off (upper trace, left), BS mode (lower traces) and BL mode (upper trace, right).

Why only BS mode works during the cycle?



Examples of beam spectrum during FT cycle

After transition
Beam intensity 4.2×10^{13}



At the end of the cycle
Beam intensity 1.6×10^{13}



Beam spectrum from 0 to 2 GHz. Horizontal scale
200 MHz/div, vertical scale linear.

Long bunches (2.5 - 3 ns).

Coupled bunch instability spectra

We consider an accelerator with M identical and equally spaced bunches. From equations of motion plus linearised Vlasov equation (A.N.Lebedev (1968), F.Sacherer (1973), J.L.Laclare (1980)) for short bunches and single multipole m excited we have

$$\frac{\Omega - m\omega_{s0}}{m\omega_{s0}} j_k = A \sum_{k'} g_{kk'}^m \frac{Z_{k'}}{k'} j_{k'}, \quad (1)$$

where $k = n + lM$, $k' = n + l'M$, $-\infty < l, l' < \infty$, $Z_k = Z(k\omega_0 + \Omega)$, $j_k = j(k\omega_0 + \Omega)$ is the Fourier transform of the beam current perturbation

$$\begin{aligned} j(\theta, t) &= e^{i\Omega t} \int_{-\infty}^{\infty} j(\omega) e^{-i\frac{\omega}{\omega_0}\theta} \frac{d\omega}{\omega_0}, \\ A &= -i \frac{J_A}{V_0 h \cos(h\theta_s) S}, \\ g_{kk'}^m &= \int_0^{\infty} \frac{d\mathcal{F}}{dr} J_m(kr) J_m(k'r) dr. \end{aligned} \quad (2)$$

Here J_A is the average beam current, V_0 is the RF voltage amplitude, $h\theta_s$ is the synchronous phase, h is the RF harmonic number, S is the normalization and $\mathcal{F}(r)$ is the unperturbed distribution function.

For the narrow band impedance with $\Delta\omega_r \ll M\omega_0$

$$\lambda j_k = A g_{kp}^m \frac{Z_p}{p} j_p \quad (3)$$

- The coherent frequency shift is

$$\lambda = \frac{\Omega - m\omega_{s0}}{m\omega_{s0}} = -i \frac{J_A}{V_0 h \cos(h\theta_s) S} g_{pp}^m \frac{Z_p}{p}. \quad (4)$$

- The unstable spectrum for coherent mode (m, n) consists of lines at frequencies

$$\omega_k = 2\pi f_k = (n + lM)\omega_0 + m\omega_s, \quad -\infty < l < \infty.$$

Negative frequencies appear on the spectrum analyzer at

$$(l + 1)M\omega_0 - n\omega_0 - m\omega_s, \quad 0 < l < \infty.$$

- The amplitudes are defined by eigenfunctions

$$j_p = 1, \quad j_k = g_{kp}^m / g_{pp}^m.$$

- For the binomial family of distribution functions

$$\mathcal{F}(r) = \mathcal{F}_0 \left(1 - \frac{r^2}{r_{max}^2}\right)^\mu$$

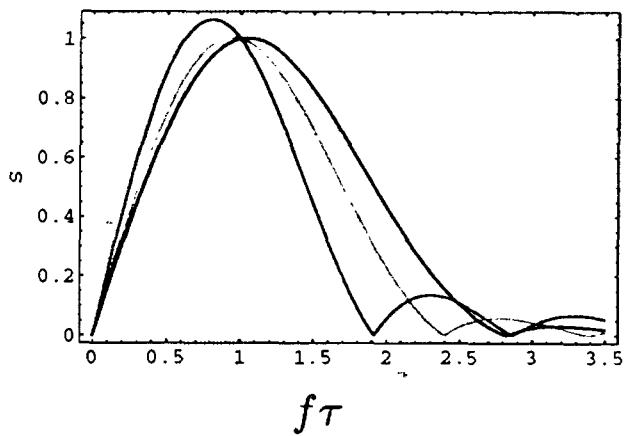
beam spectrum envelope is

$$j_k \propto \int_0^1 x(1-x^2)^{\mu-1} J_m(y_k x) J_m(y_r x) dx, \quad (5)$$

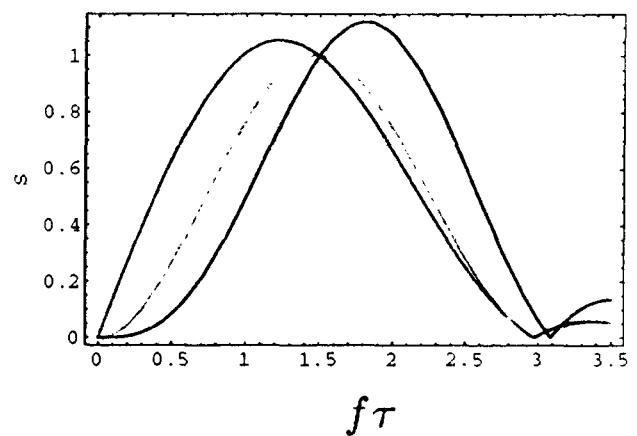
where $y_k = kr_{max} \simeq \pi f_k \tau$ and $y_r = p_r r_{max} \simeq \pi f_r$.

Spectrum envelope for coupled-bunch modes

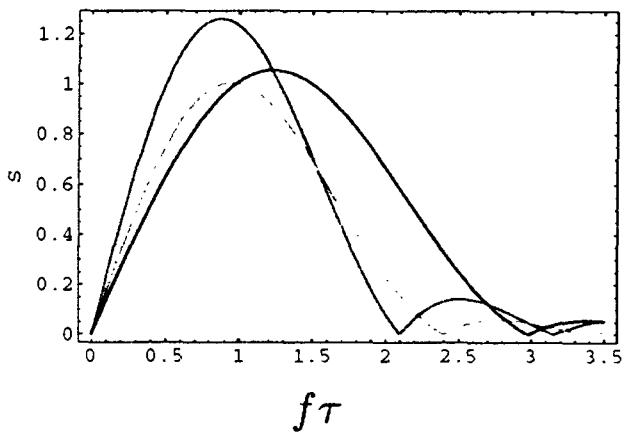
$\mu = 1.5, 2, 2.5, m=1, f_r\tau=1$



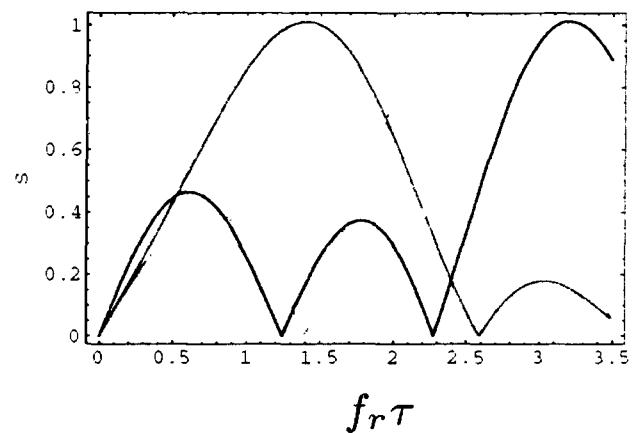
$m = 1, 2, 3, \mu = 2 f_r\tau=1.5$



$f_r\tau=0.5, 1.0, 1.5, m=1, \mu = 2$



$m = 1, \mu = 1, f_r\tau = 1.5, \dots, 3$



Particle distribution: $\lambda(t) = \lambda_0(1 - \frac{4t^2}{\tau^2})^{\mu+0.5}$.

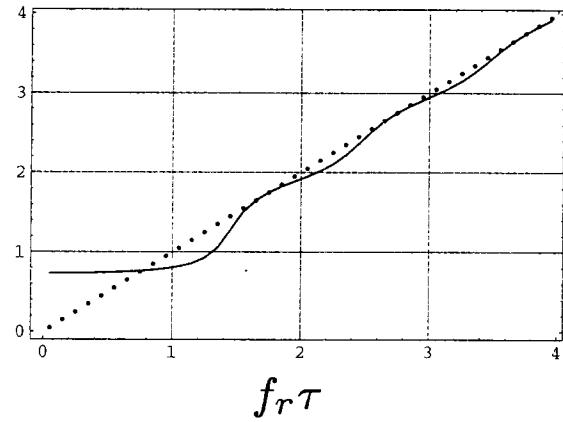
Spectrum:

$$j_k \sim \int_0^1 x(1 - x^2)^{\mu-1} J_m(\pi f_r \tau x) J_m(\pi f_k \tau x) dx.$$

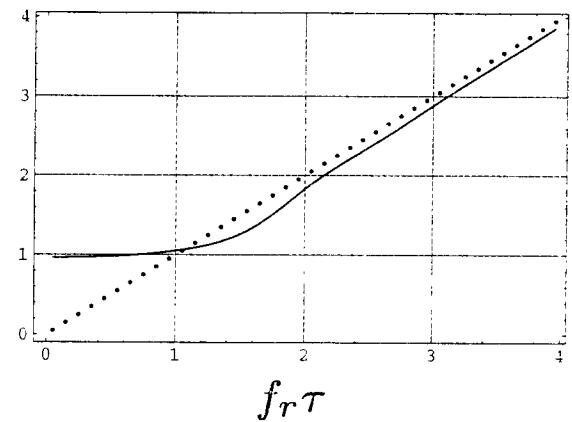
Position of the maximum

in the beam spectrum envelope

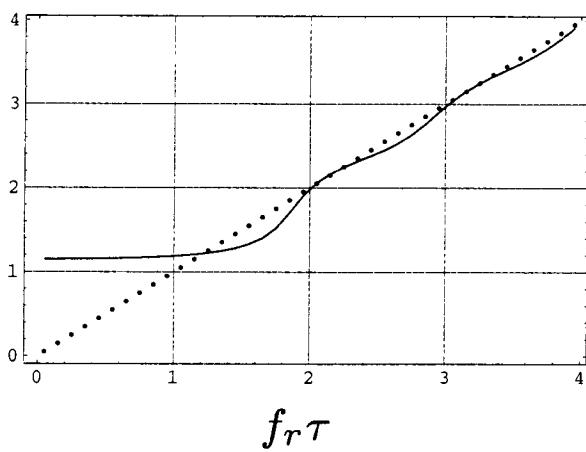
$f_{max}\tau \text{ m=1, } \mu = 1$



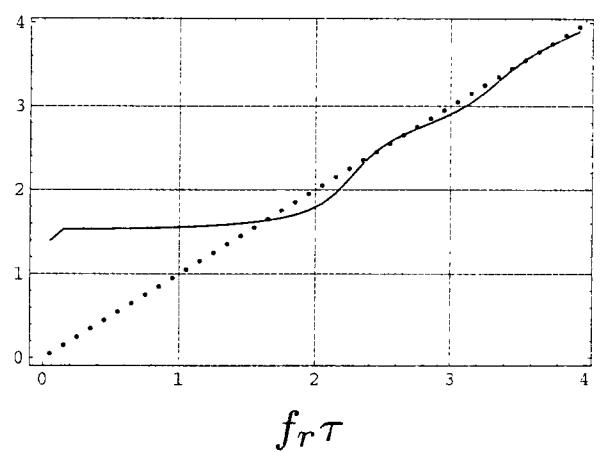
$f_{max}\tau \text{ m=1, } \mu = 3$



$f_{max}\tau \text{ m=2, } \mu = 1$



$f_{max}\tau \text{ m=3, } \mu = 1$



Two regimes:

$$f_r\tau < 1 \longrightarrow f_{max}\tau = \text{const}(m, \mu)$$

$$f_r\tau > 1 \longrightarrow f_{max} \simeq f_r$$

The existence of two different regimes can formally be understood from the behaviour of the Bessel functions

- For $f_r \tau < 1$

$$j_k \propto \int_0^1 x(1-x^2)^{\mu-1} x^m J_m(y_k x) dx$$

For $\mu = 1$ the beam spectrum has amplitudes

$$j_k \propto \frac{J_{m+1}(y_k)}{y_k}.$$

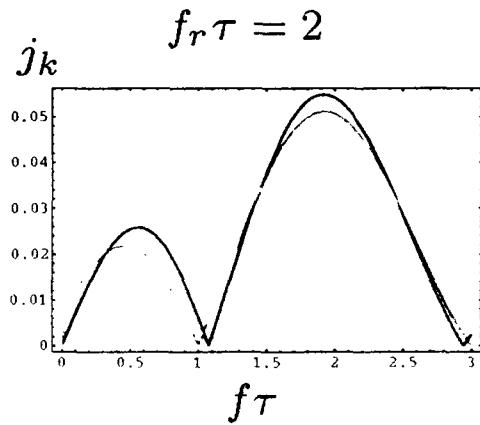
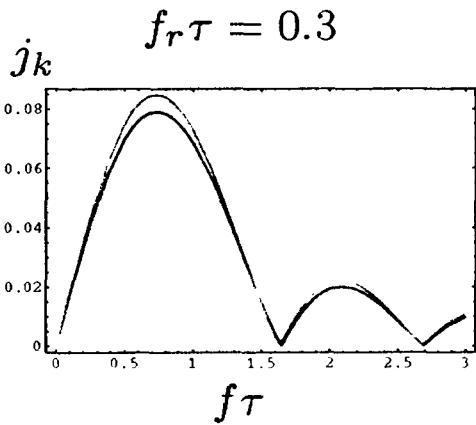
- For $f_r \tau > 1$ and $f_k \tau > 1$

$$j_k \sim \frac{1}{y_k^{1/2}} \int_0^1 (1-x^2)^{\mu-1} \cos[(y_k - y_r)x] dx.$$

For $\mu = 1$ this gives

$$j_k \sim \frac{\sin(y_k - y_r)}{y_k^{1/2} (y_k - y_r)}.$$

Approximate and exact solutions for beam spectrum envelope for $m = 1$ and $\mu = 1$



The existence of two different regimes can formally be understood from the behaviour of the Bessel functions

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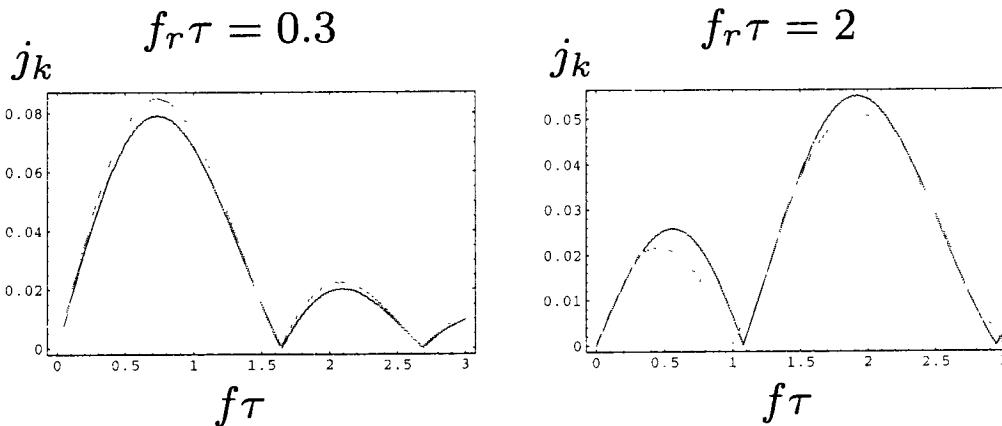
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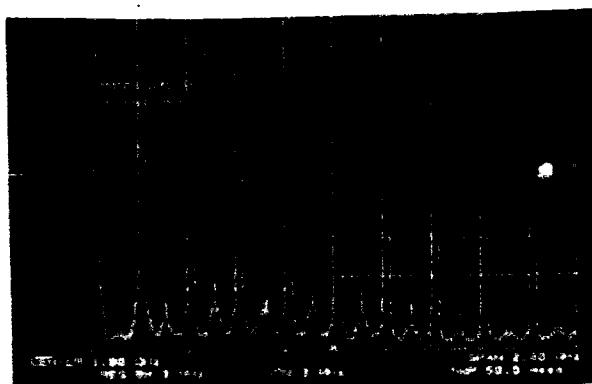
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Approximate and exact solutions for beam spectrum envelope for $m = 1$ and $\mu = 1$



Spectrum of low intensity fixed target beam

$$N = 3.9 \times 10^{12}$$



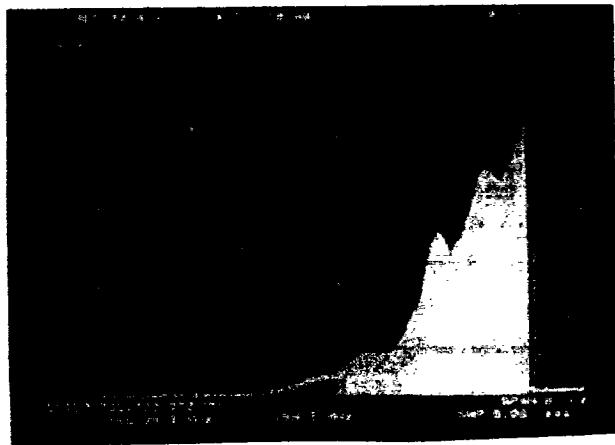
0

2 GHz

0

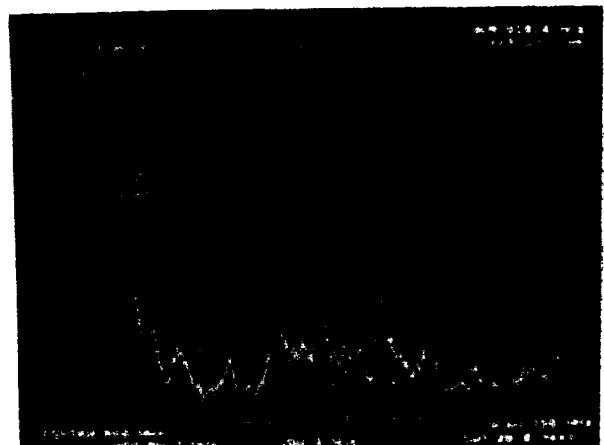
2 GHz

Signal in time at 705 MHz



time

Sidebands around 900 MHz



800 MHz

1 GHz

$$f_{max} \simeq 700 \text{ MHz}, \tau \sim (1.5 - 2) \text{ ns} \rightarrow f_{max}\tau = 1 - 1.4$$

$$nf_0 \simeq 113 \text{ MHz} \rightarrow \text{HOM of TW200? } f_r = 912 \text{ MHz}$$

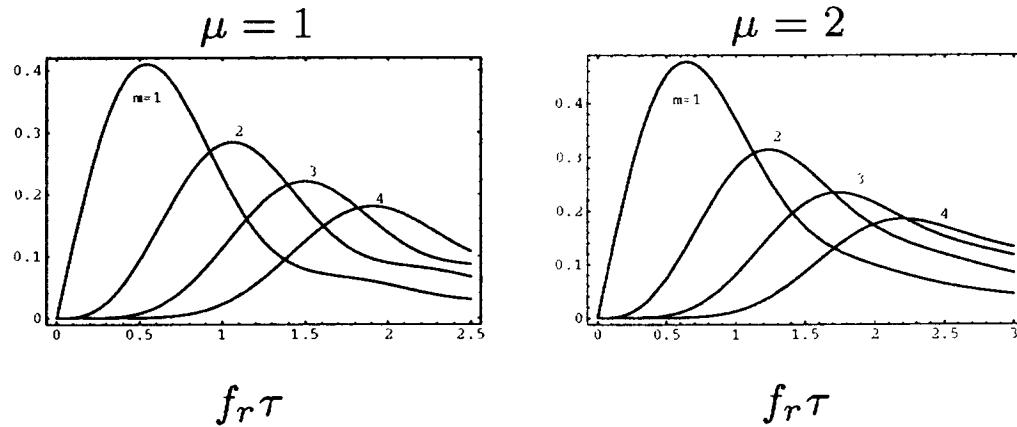
Growth rates

$$\frac{\text{Im}\Omega}{\omega_s} = \frac{4}{\pi^2} \frac{J_A \text{Re}Z}{hV_0 \cos(h\theta_s)} \frac{F_m^*}{f_0\tau},$$

where for the binomial distribution function:

$$F_m^* = \frac{m\mu(\mu+1)}{f_r\tau} \int_0^1 x(1-x^2)^{\mu-1} J_m^2(y_r x) dx.$$

Formfactor F_m^* as a function of $f_r\tau$ for $m=1,2,3,4$



For $\mu = 1$, mode $m = 1$ is dominant up to $f_r\tau \simeq 0.9$,
 $m=2$: $0.9 - 1.4$, $m=3$: $1.4 - 1.85\dots$

Summary for measured beam spectra analysis:

- $f_{max}\tau < 1 \rightarrow f_r\tau \leq 1.2$,
- $f_{max}\tau > 1 \rightarrow f_r \sim f_{max}$.

The uncertainty does not exceed $\pm 0.2/\tau$. The smaller the bunch length - the larger the uncertainty.

Conclusions → future plans

- Study of stability of the LHC beam in the SPS
 - optimization of voltage programme
 - emittance measurements at minimum bunch length
- Identification of sources of coupled-bunch instability for fixed target beam
- Continued development of cures for instabilities
- ...